

# **Aristotle's Patterns Of Logic and Ontology**

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# Foundations of Ontology

## Hypotheses proposed by the pre-Socratic philosophers:

- **Pythagoras:** Mathematical forms govern everything.
- **Heraclitus:** An eternal flux (*panta rhei*) governed by laws (*logos*).
- **Empedocles:** Four elements (fire, air, water, earth) and two principles of change: love (*eros*) and strife (*eris*).
- **Leucippus and Democritus:** Tiny atoms in constant motion.

## Disciplined methods for analyzing hypotheses:

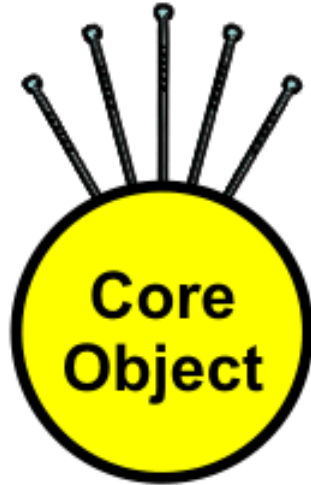
- **Socrates:** Conceptual analysis by systematic questioning.
- **Plato:** Theories presented in dialogs led by Socrates.
- **Aristotle:** Systematic treatises instead of free-flowing dialogs.

## Aristotle's theories set the standard for logic and ontology:

- **Stable objects as composites of form and matter.**
- **Ten categories for analyzing, describing, and classifying anything.**
- **Logic for specifying patterns and reasoning about them.**

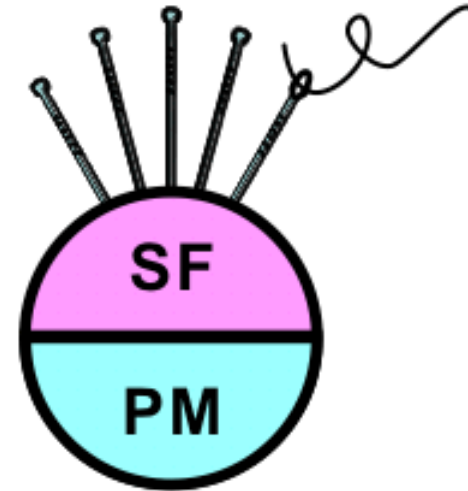
# Differences Between Plato and Aristotle

Properties



Plato's Ontology

Accidents



Aristotle's Ontology

Paul Spade drew “pincushion diagrams” to illustrate ontologies.

For Plato, each object is represented as a bundle of properties (pins) contained in a receptacle (the pincushion).

The pins for optional properties can be removed or replaced.

But the pins for essential (necessary) properties have a barbed hook that cannot be removed without destroying the pincushion.

# Aristotle's Ontology

The pincushions for Aristotle's ontology contain an unchangeable substantial form (SF) and changeable prime matter (PM).

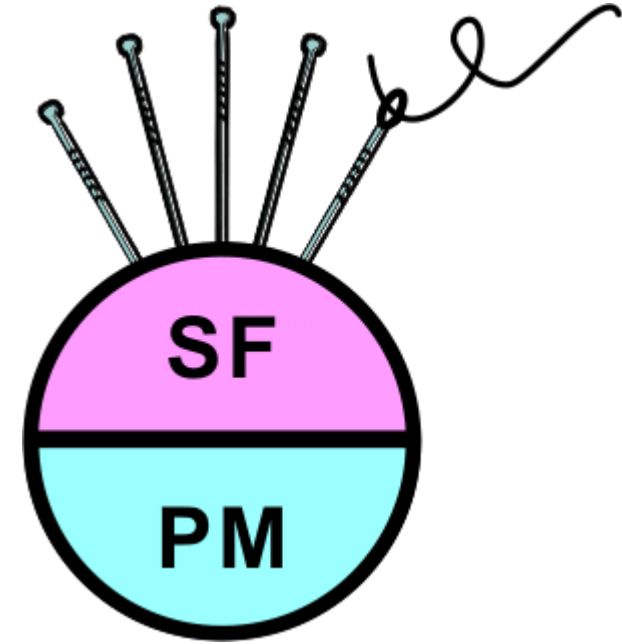
All pins represent *accidents* that may be removed or replaced.

For relations, the pins resemble needles with threads linked to the needles of other objects.

Although all accidents are changeable, some kinds of accidents are necessarily present.

For example, all physical objects must have weight, but the weight can change as the amount of matter changes.

For such accidents, the SF determines preassigned slots that must always be filled with some pin of the appropriate type.



# Aristotle's Categories

**Ten ways of describing anything that exists or can exist.**

**Each category has a corresponding question:**

- **Substance – What is it?**
- **Relation – Toward what?**
- **Quantity – How much?**
- **Quality – What kind?**
- **Activity – Doing what?**
- **Passivity – Undergoing what?**
- **Condition – Having what?**
- **Position – How situated?**
- **Place – Where?**
- **Time – When?**

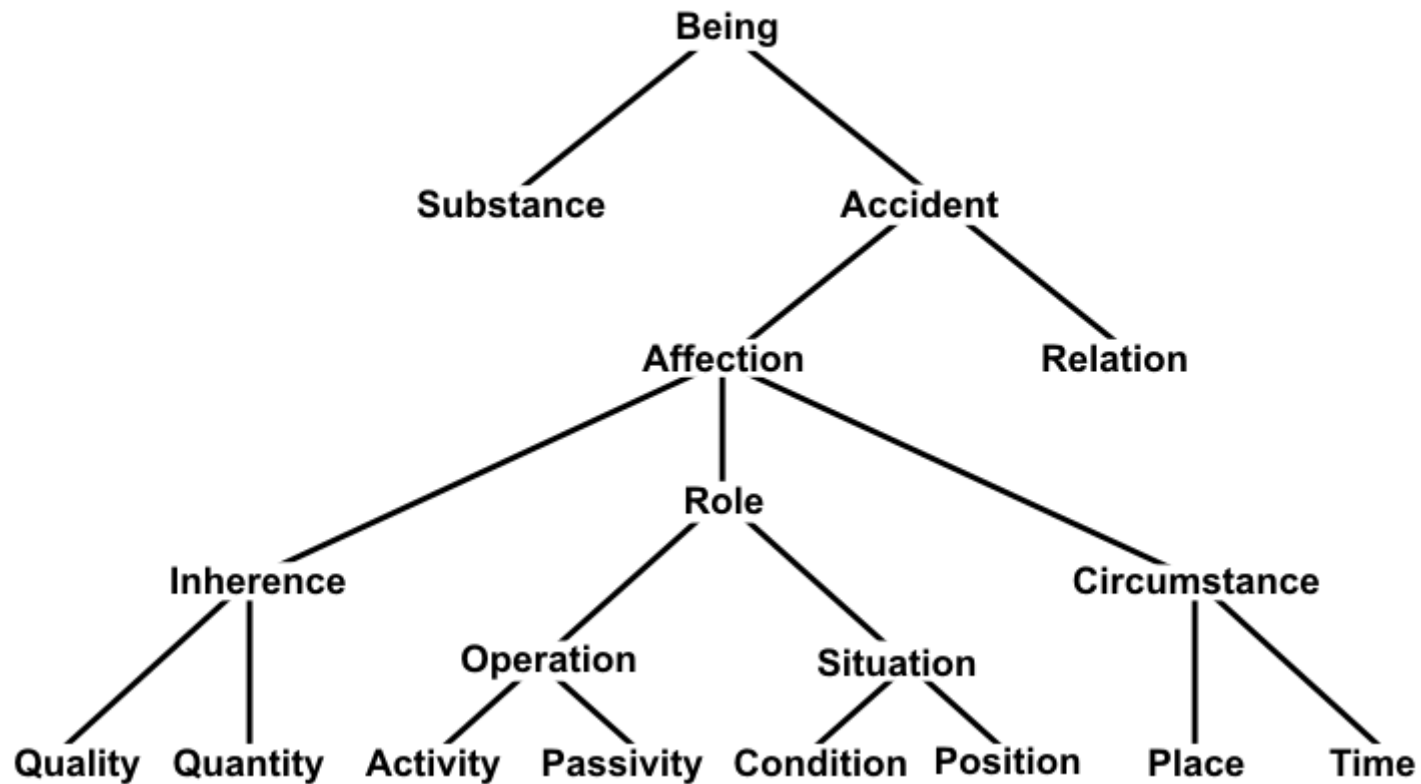
**Substance is the unchanging form that determines what something is.**

**The other nine categories describe accidents that can change.**

# Describing George Washington

Category	Question	Answer
Substance	What is it?	A man
Relation	Toward what?	A general of the US army
Quality	What kind?	Handsome
Quantity	How much?	Tall
Activity	Doing what?	Talking
Passivity	Undergoing what?	Listening
Condition	Having what?	Victory in battle
Position	How situated?	Mounted on a horse
Place	Where?	Yorktown, Virginia
Time	When?	19 October 1781, 2 pm

# Tree of Aristotle's Categories



**Aristotle's categories, as arranged by Franz Brentano (1862).**

**The ten categories are the endpoints (leaves) of the tree.**

**The branch points are based on writings by Aristotle.**

# Aristotle's Pattern for Definitions

Define categories by *genus* and *differentiae*:

- Genus – a supertype of the category to be defined.
- Species – the subtype that is being defined.
- Differentia – a property or attribute that distinguishes the species to be defined from other species of the same genus.

This pattern is still the standard for a good definition.

Ideally, the differentiae should state necessary and sufficient conditions that uniquely distinguish the species.

That goal is achieved in mathematics.

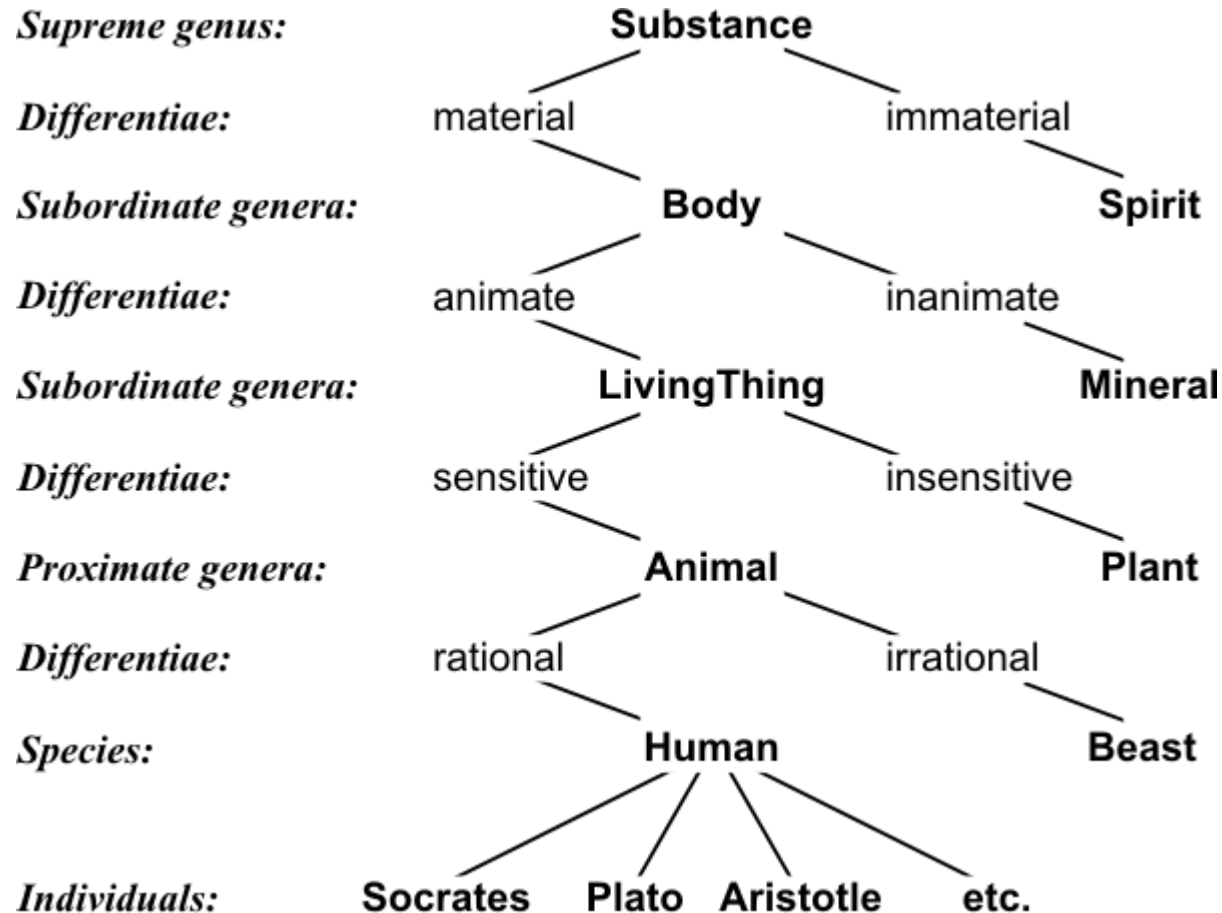
But it is rarely possible for most artifacts and natural kinds.

As a discovery method in biology, Aristotle used an alternative:

- For each species, describe a type specimen in detail.
- Use looser criteria of similarity to determine the genus.



# Tree of Porphyry



**A tree of categories and differentiae was found in a commentary on Aristotle by the philosopher Porphyry (3<sup>rd</sup> century AD).**

**This tree is based on a version by Peter of Spain (1239).**

# Body and Soul

All the Greeks used the word *psychê*, which is translated as *anima* in Latin and *soul* in English.

Plato assumed a separable psyche, which uses the body as the instrument of perception and action.

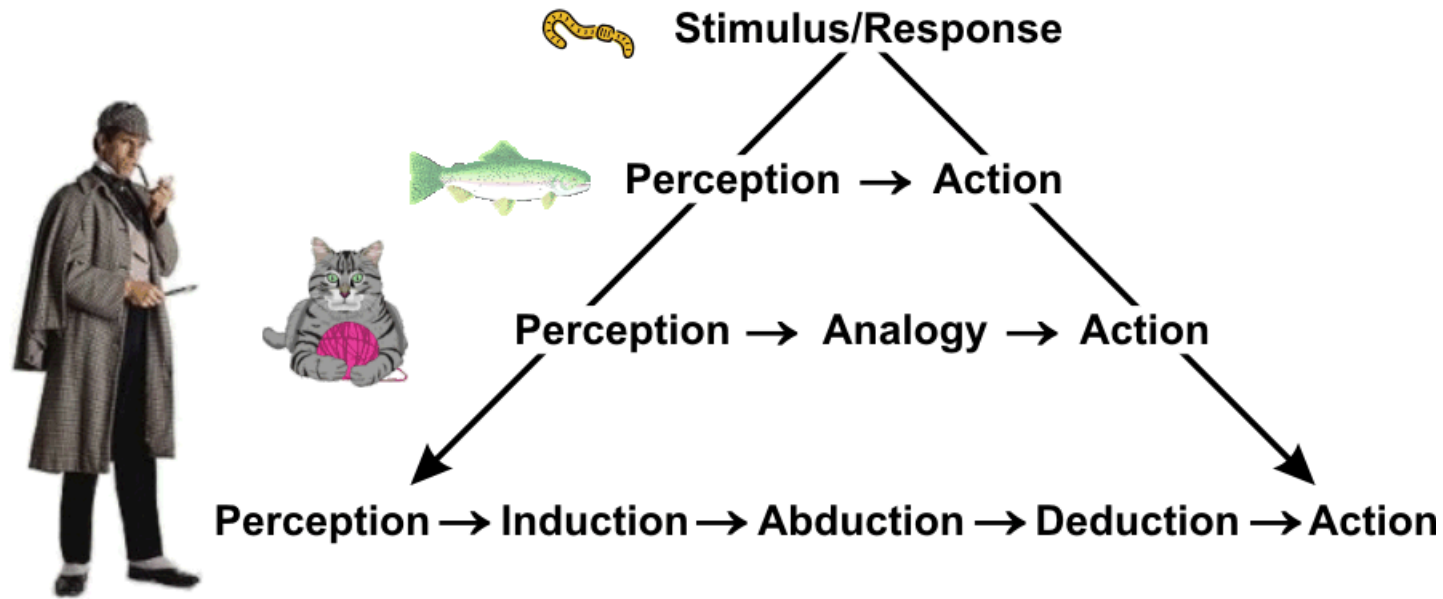
But Aristotle assumed that every living thing has an embodied psyche, which serves as its substantial form:

- Nutritive psyche for plants.
- Sensitive psyche for sessile animals like sponges and clams.
- Locomotive psyche for worms.
- Psyche with imagery for animals with eyes.
- Rational psyche for humans.

The psyche controls all growth and motion from birth to death.

The psyches of the more advanced animals inherit all the functions of the more primitive psyches.

# Cognitive Complexity



**Each level inherits the abilities of all previous levels.**

**Aristotle's levels of the psyche are compatible with evolution.**

**They are also compatible with the biblical account in Genesis.**

**Being omniscient, God wisely chose the evolutionary method of system design and development.**

# Experimental Science

**Plato was a mathematician who emphasized the unchangeable mathematical forms as the most reliable source of knowledge.**

**But Aristotle's father was a physician, from whom he learned the need for detailed observation and careful records.**

**Although Aristotle is famous for his innovations in logic, his most voluminous writings are on biology.**

**He and his students carried out a classic experiment:**

- Select 30 chicken eggs laid on the same day. \***
- Break open one egg each day, write a detailed description of the embryo, and analyze the stages of growth.**

**Plato would never ask his students to do such experiments.**

**As late as the 17<sup>th</sup> century, the physician William Harvey said Aristotle's writings on embryology were still unsurpassed.**

**\* The incubation period is 21 days, but it's good to have extra eggs to allow for breakage.**

# Modes of Explanation

**Aristotle defined four modes or principles for explaining change.**

**His Greek word *aitia* was translated to Latin as *causa*, but the English word *cause* has shifted in meaning.**

**Aristotle's four aitia:**

- **Efficient:** Something that initiates a change or carries it through.
- **Material:** The matter or resources used in the change.
- **Formal:** The basic form, plan, or pattern of the change.
- **Final:** The goal, purpose, or function (in Greek, *telos*).

**The English word *cause* is usually limited to the efficient aitia.**

**In biological explanations, Aristotle emphasized the function (final aitia or telos) of any organ or feature of a living thing.**

**This principle is still important for understanding how and why living things and their parts grow and operate.**

# Syllogisms

**Aristotle coined the word *syllogism* for any valid pattern of inference. Today, they're called rules of inference.**

**Three kinds of syllogisms developed in antiquity:**

- **Categorical syllogisms for reasoning about categories.**
- **Hypothetical syllogisms based on *if-then* patterns.**
- **Disjunctive syllogisms based on inclusive and exclusive *or*.**

**All these patterns (and many more) are still important.**

**In fact, the majority of OWL ontologies today use only the basic patterns that Aristotle defined.**

**But Aristotle used a more readable notation based on a controlled natural language (CNL).**

# Patterns that Relate Categories

Four sentence patterns used in categorial syllogisms:

<b>A</b>	<b>Universal affirmative:</b>	<b>Every S is P.</b>
<b>I</b>	<b>Particular affirmative:</b>	<b>Some S is P.</b>
<b>E</b>	<b>Universal negative:</b>	<b>No S is P.</b>
<b>O</b>	<b>Particular negative:</b>	<b>Some S is not P.</b>

Boethius (6<sup>th</sup> century AD) introduced the letters A, I, E, and O as mnemonics for naming and remembering the combinations:

- A and I are the first two vowels in the Latin *affirmo* (I affirm).
- E and O are the first two vowels in *nego* (I deny).

The letters S and P represent *terms* in the subject and predicate.

Each term is a word that names a category or a phrase that states some *differentia* for defining a category.

# Categorical Syllogisms

**Barbara, the name of the first pattern, has three As, which indicate three type A sentences:**

- A Every animal is a living thing.**
- A Every human is an animal.**
- A Therefore, every human is a living thing.**

**The pattern named Darii has sentence types A, I, I:**

- A Every beast is irrational.**
- I Some animal is a beast.**
- I Therefore, some animal is irrational.**

**The pattern Barbara supports the inheritance of properties from a supertype to every individual of a subtype.**

**The pattern Darii supports the inheritance of properties from a type to particular individuals of a different type.**



# Negative Syllogisms

The first negative syllogism is named Celarent:

- E** No body is a spirit.
- A** Every animal is a body.
- E** Therefore, no animal is a spirit.

The negative syllogism Ferio applies to particular individuals:

- E** No plant is rational.
- I** Some living thing is a plant.
- O** Therefore, some living thing is not rational.

**E** and **O** sentences express constraints on the type hierarchy.

Negative syllogisms derive the implications of those constraints.

# Syllogisms with Other Verbs

Sentences with the verb *has* and an arbitrary object:

- A Every quadruped has four legs.
- A Every elephant is a quadruped.
- A Therefore, every elephant has four legs.

But any verb or verb phrase can be used:

- A Every mammal breathes oxygen.
- I Some sea creature is a mammal.
- I Therefore, some sea creature breathes oxygen.

With a suitable paraphrase, all A, I, E, O sentences can be converted to a pattern with *is* as the main verb:

- Every quadruped is an animal with four legs.
- Every mammal is an oxygen-breathing animal.

# Other Paraphrases

The pattern Cesare has the same predicate in both premises:

- E No oyster has lungs.
- A Every mammal has lungs.
- E Therefore, no mammal is an oyster.

With a paraphrase, Cesare can be converted to Celarent:

- E No animal with lungs is an oyster.
- A Every mammal is an animal with lungs.
- E Therefore, no mammal is an oyster.

Negating some verbs requires the auxiliary verb *does*:

- E No herbivore eats meat.
- I Some mammal is a herbivore.
- O Therefore, some mammal does not eat meat.

# Patterns of Categorical Syllogisms

Each categorical sentence affirms or denies a type-subtype relationship between two categories.

The verb *is* links terms that may be subjects or predicates.

Terms that use other verbs may only occur as predicates.

- A Every quadruped has four legs.
- I Some sea creature breathes oxygen.
- E No oyster has lungs.
- O Some mammal does not eat meat.

There are 256 possible patterns of 3-sentence syllogisms.

But only 19 of those patterns are logically valid.

The other 237 patterns are fallacious: they could derive a false conclusion from true premises.

# The 19 Valid Patterns

**Aristotle chose Barbara and Celarent as the two basic patterns.**

**The other 17 valid patterns are derived by conversion rules.**

**Darii is derived from Barbara, and Ferio from Celarent.**

**Barbara, Celarent, Darii, and Ferio belong to the *first figure*.**

**The other valid patterns belong to figures 2, 3, and 4:**

**2. Cesare, Camestres, Festino, Baroco.**

**3. Darapti, Felapton, Disamis, Datisi, Bocardo, Ferison.**

**4. Bamalip, Calames, Dimatis, Fesapo, Fresison.**

**Patterns whose name begins with B are derived from Barbara; with C, from Celarent; with D, from Darii; with F from Ferio.**

**The consonants after the vowels indicate the conversion rules.**

# Patterns of the Four Figures

$$\begin{array}{l} M - P \\ S - M \\ \hline \therefore S - P \end{array}$$

Figure 1

$$\begin{array}{l} P - M \\ S - M \\ \hline \therefore S - P \end{array}$$

Figure 2

$$\begin{array}{l} M - P \\ M - S \\ \hline \therefore S - P \end{array}$$

Figure 3

$$\begin{array}{l} P - M \\ M - S \\ \hline \therefore S - P \end{array}$$

Figure 4

The letters **S**, **P**, and **M** represent terms:

- **S** is the term that appears in the subject of the conclusion.
- **P** is the term in the predicate of the conclusion.
- **M** is the middle term. It appears in the premises, but not the conclusion.

For example, Felapton belongs to the third figure.

It would have **E**, **A**, **O** sentences in the following pattern:

No **M** is **P**.

All **M** is **S**.

Therefore, some **S** is not **P**.

The name and the figure determine the pattern of sentences.

# Propositional Logic

**Aristotle made some comments about propositional logic.**

**He even used variables to represent propositions:**

- **“If when A is, B must be, then when B is not, A cannot be.”**
- **In Latin, this rule is called *modus tollens* (mode of taking away).**

**The Stoic philosophers developed propositional logic further.**

**In his commentaries, Boethius summarized the Stoic logic.**

- **When a formula mentions two propositions, Boethius used the Latin *hoc* (this) for the first and *illud* (that) for the second.**
- **But when a formula requires more than two, he used letters.**
- **Example: “Sicut cum A est, B est, C est.”  
If (when A is, B is), C is.**
- **In symbolic logic:  $(A \supset B) \supset C$ .**
- **The old Latin names are still used for the inference rules.**

# Hypothetical Syllogisms

A hypothetical syllogism uses *if-then* rules with arbitrary sentence patterns and two rules of inference.

The rule *modus ponens* (mode of putting) makes an assertion:

Given	If A, then B.
But	A.
Therefore	B.

The rule *modus tollens* (mode of taking away) denies the if-part:

Given	If A, then B.
But	not B.
Therefore	not A.

In modern rule-based systems, a deduction by repeated modus ponens is called *forward chaining*.

Repeated modus tollens is called *backward chaining*.



# Hypothetical and Disjunctive Syllogisms

If p, then q.  
But p.

---

∴ q.

Modus ponens

If p, then q.  
But not q.

---

∴ not p.

Modus tollens

p or q.  
But not p.

---

∴ q.

Inclusive or

p xor q.  
But p.

---

∴ not q.

Exclusive or

p xor q.  
But not p.

---

∴ q.

Exclusive or

The word *or* is ambiguous in natural languages.

The Stoic philosophers recognized that ambiguity.

They defined one pattern for inclusive or; two for exclusive or.

By convention, the Latin logicians used *vel* for inclusive or and the pattern *aut ... aut ...* for exclusive or.

But in ordinary usage, the Latin *aut ... aut ...* was just as ambiguous as the English *either ... or ...*

# Disjunctive Syllogisms

A disjunctive syllogism is based on *either-or* patterns.

The basic disjunctive syllogism allows arbitrary sentences:

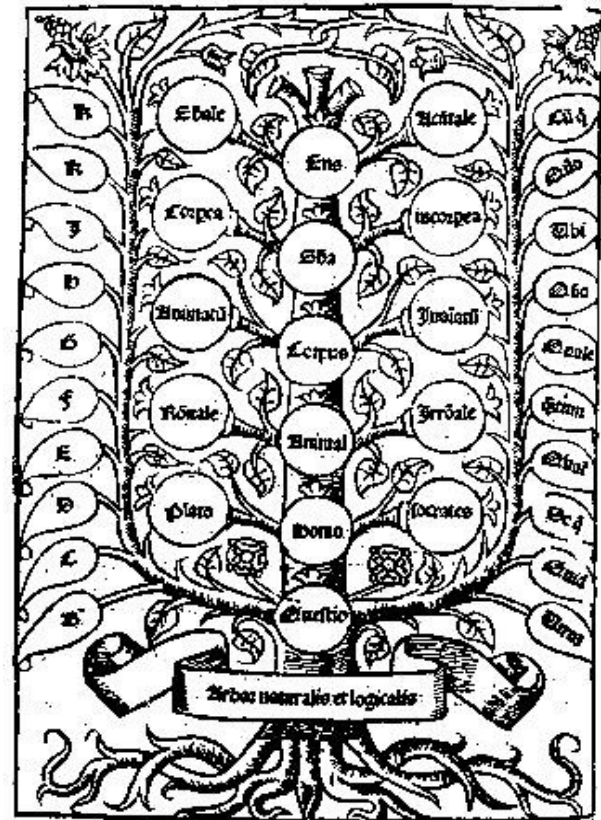
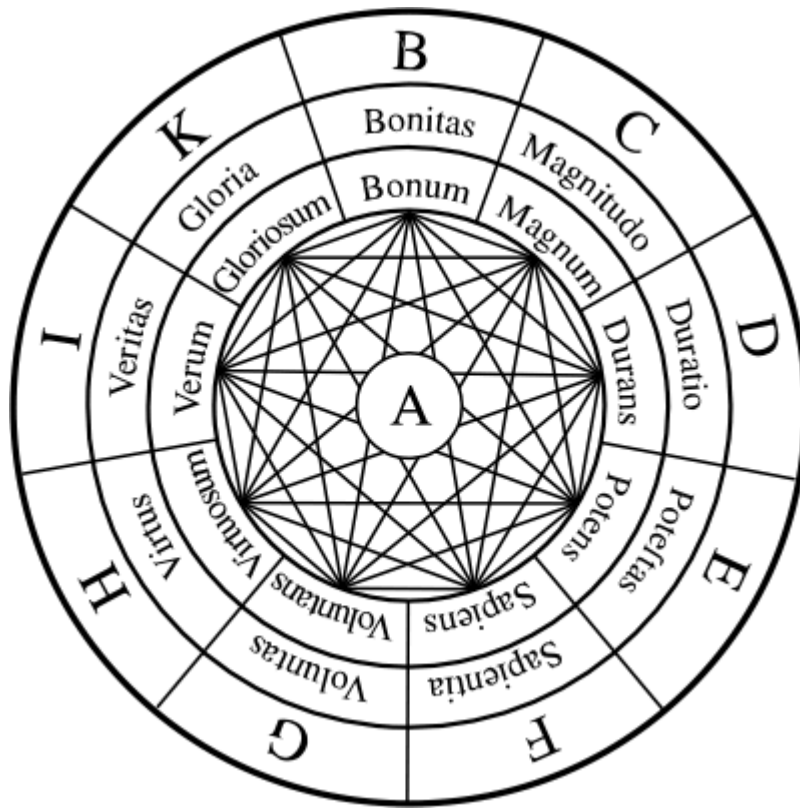
Given	Either A or B.
But	not A.
Therefore	B.

A disjunctive syllogism can also be applied to the predicate of a type I sentence, as in the following argument:

Given	Every philosopher is either sleeping or debating.
But	Socrates is a philosopher.
Therefore	Socrates is either sleeping or debating.
But	Socrates is not sleeping.
Therefore	Socrates is debating.

Note that a disjunctive syllogism cannot be applied to the first premise because it is a type A sentence.

# Circles and Diagrams by Ramon Llull



Llull was a Catalan poet, philosopher, and missionary. He developed a system of rotating circles for combining attributes and relating them to Aristotle's categories. It was an early device for mechanical reasoning.

# Gottfried Wilhelm Leibniz

**Leibniz was a mathematician, philosopher, and diplomat who made important contributions to ontology:**

*“The art of ranking things in genera and species is of no small importance and very much assists our judgment as well as our memory... And those who have laid out all sorts of notions under certain headings or categories have done something very useful.” \**

**Leibniz was also inspired by Lull’s combinatorial method.**

**For his *Universal Characteristic*, he assigned a prime number to each differentia and products of primes to each category.**

**Category A is more general than category B iff the integer for A divides the integer for B.**

**The result is a system of multiple inheritance called a *lattice*.**

# Encoding the Tree of Porphyry

**Assign 1 to Substance, the supreme genus.**

**Assign prime numbers to the differentiae:**

**material 2; immaterial 3; animate 5; inanimate 7; sensitive 11; insensitive 13; rational 17; irrational 19.**

**The number for each category is the product of the number for its genus times the number for its differentia:**

**Body is  $1 \times 2 = 2$ . Spirit is  $1 \times 3 = 3$ .**

**LivingThing is  $2 \times 5 = 10$ . Mineral is  $2 \times 7 = 14$ .**

**Animal is  $10 \times 7 = 70$ . Plant is  $10 \times 13 = 130$ .**

**Human is  $70 \times 17 = 1190$ . Beast is  $70 \times 19 = 1330$ .**

**Category A is more general than category B iff the number for A divides the number for B.**

# Calculating Machine by Leibniz

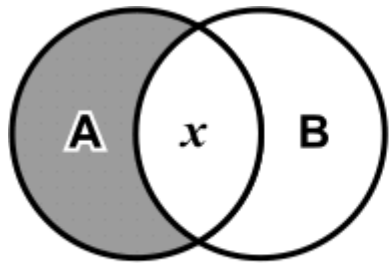


**Leibniz invented the first calculating machine that could do multiplication and division.**

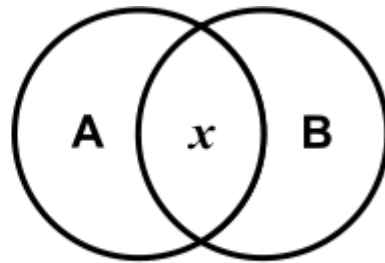
**He used it for mechanical reasoning about patterns of relations in the numeric encoding of his categories.**

**His method of using prime numbers inspired Kurt Gödel, who used prime numbers to encode patterns of logic.**

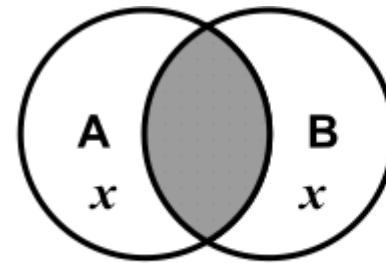
# Venn Diagrams



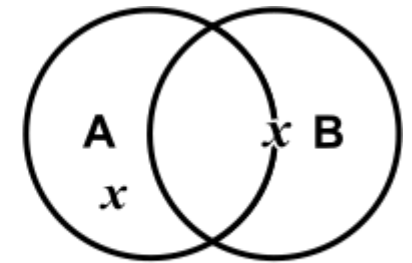
Every A is B



Some A is B



No A is B



Some A is not B

In the 19<sup>th</sup> century, John Venn used patterns of overlapping circles to determine the valid patterns of syllogisms.

Each circle contains every instance  $x$  for which some A is true.

Four kinds of areas:

- Shaded area: known to be empty – no instances exist.
- White area: contents unknown – some instance  $x$  might exist.
- White area marked with  $x$ : contains at least one instance  $x$ .
- Border marked with  $x$ : some  $x$  exists on one side or the other.

Note: Aristotle assumed that every category has at least one  $x$ .

# Venn Diagrams for Barbara and Darii

For the pattern Barbara, the area of A outside B is empty, and the area of C outside A is empty.

For Darii, the area of A outside B is empty, but the area of C outside A is irrelevant.

Therefore, any data that makes Barbara true will also make Darii true.

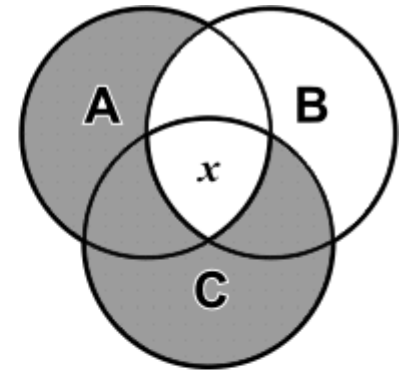
This observation shows that Aristotle's derivation of the pattern Darii from Barbara is valid: it preserves truth.

Exercise: Draw Venn diagrams to show how patterns of the second, third, and fourth figures can be derived from the first.

Every A is B.  
Every C is A.  

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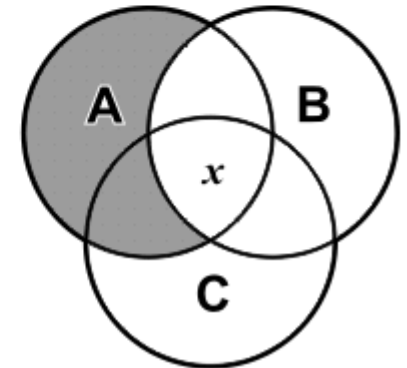
∴ Every C is B.



Every A is B.  
Some C is A.  

---

∴ Some C is B.





# Model Theory

**Alfred Tarski (1933) developed model theory for first-order logic, but Venn invented an equivalent model theory for syllogisms.**

**For first-order logic, a Tarski model consists of**

- A set  $D$  of individuals in the domain of discourse.**
- For each  $N$ -adic relation, a set of  $N$ -tuples of individuals in  $D$  for which the relation is true.**
- A method for evaluating the truth of any FOL statement in terms of the set  $D$  and the sets of  $N$ -tuples.**

**For syllogisms, Venn's method is equivalent to Tarski's:**

- A set  $D$  of individuals in the domain of discourse.**
- For each monadic relation, a circle that represents the set of individuals (1-tuples) in  $D$  for which the relation is true.**
- A method for evaluating the truth of any A, I, E, O statement in terms of the set  $D$  and the sets of 1-tuples.**

# Observations

**Aristotelian-Stoic-Scholastic logic was a brilliant achievement.**

**It is the foundation for *description logics* such as OWL.**

**The methods of conceptual analysis are essential for all work in ontology, semantics, and knowledge representation.**

**But traditional logic is not sufficiently expressive:**

- **The controlled NL for syllogisms is a small subset of full NL.**
- **Euclid needed a larger subset for his textbook on mathematics.**
- **Computer science requires full first-order logic – or more.**

**The patterns of syllogisms are useful, but the conversion rules are too complex.**

**Venn diagrams show that simple but general patterns can make a complex subject easier to learn, remember, and use.**

# Patterns of Modern Logic

## Several kinds of syntax:

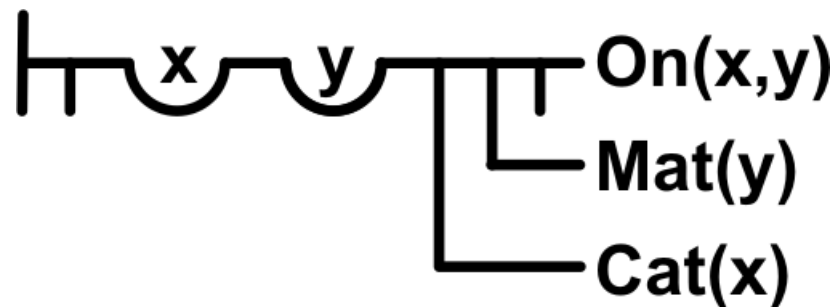
- **Algebraic:** Boolean algebra and predicate calculus.
- **General purpose diagrams:** Frege's trees, Peirce's graphs.
- **Special purpose diagrams:** Type hierarchies, UML diagrams.
- **Controlled natural languages.**
- **Miscellaneous:** Mixed notations for a variety of purposes.

## Design choices:

- **Optimize readability.**
- **Optimize expressive power.**
- **Tailor the notation for the reasoning method.**
- **Tailor the notation for special kinds of applications.**
- **Mix some special-purpose ontology with the logic.**

# Notations for Saying “*A cat is on a mat.*”

Frege (1879) designed a tree notation whose structure was tailored to the patterns of his axioms and rules of inference.



But nobody else adopted his notation. \*

Peirce added *quantifiers* and *relations* to Boolean algebra.

Peano adopted Peirce’s algebra, but with different symbols:

Peirce (1885):  $\Sigma_x \Sigma_y \text{Cat}_x \cdot \text{Mat}_y \cdot \text{On}_{x,y}$

Peano (1890):  $\exists x \exists y \text{Cat}(x) \wedge \text{Mat}(y) \wedge \text{On}(x,y)$

But in 1897, Peirce invented *existential graphs* as a readable notation with simple, but general rules of inference: \*\*

**Cat — On — Mat**

\* See “Peirce the Logician” by Hilary Putnam, <http://www.jfsowa.com/peirce/putnam.htm>

\*\* For an introduction to existential graphs, see <http://www.jfsowa.com/talks/egintro.pdf>

# Related Readings

Summary of notations and terminology for mathematics and logic,  
<http://www.jfsowa.com/logic/math.htm>

Slides for an introduction to Peirce's existential graphs (EGs),  
<http://www.jfsowa.com/talks/egintro.pdf>

Role of Logic and Ontology in Language and Reasoning,  
<http://www.jfsowa.com/pubs/rolelog.pdf>

Fads and Fallacies About Logic,  
<http://www.jfsowa.com/pubs/fflogic.pdf>

Peirce's tutorial on existential graphs,  
<http://www.jfsowa.com/pubs/egtut.pdf>

Conceptual graphs and Common Logic,  
[http://www.jfsowa.com/cg/cg\\_hbook.pdf](http://www.jfsowa.com/cg/cg_hbook.pdf)

ISO/IEC standard 24707 for Common Logic,  
[http://standards.iso.org/ittf/PubliclyAvailableStandards/c039175\\_ISO\\_IEC\\_24707\\_2007\(E\).zip](http://standards.iso.org/ittf/PubliclyAvailableStandards/c039175_ISO_IEC_24707_2007(E).zip)