Peirce’s Contributions to the 21st Century

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Peirce’s Contributions

In the 19th century, Peirce had an international reputation in both logic and physics.

- Logic: Invented the algebraic notation for symbolic logic.
- Astronomy: Photometric researches for measuring the distance to stars.
- Mathematics: Revised and extended his father’s book on linear algebra.
- Physics: Designed new instruments for measuring gravity.
- Engineering: First to propose a wavelength of light as a standard of length, and built the instruments for using light waves to measure the length of his pendulums.

20th century:

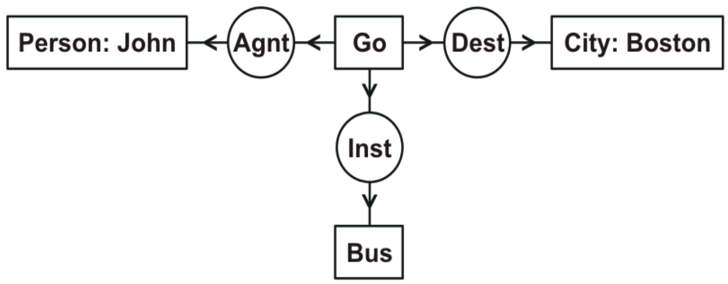
- Largely ignored by both the analytic philosophers and the so-called Continental philosophers.

Promising direction for the 21st century:

- Established a foundation for unifying and relating the insights of multiple branches of cognitive science (linguistics, philosophy, psychology, and artificial intelligence).

This talk surveys Peirce’s approaches, their applications in modern systems, and the way his views of language and logic can clarify and support a more comprehensive view of natural language semantics.

Sentence: *John is going to Boston by bus*



The conceptual graph can be translated to other notations for logic by mapping the conceptual relations **(Agnt)** for agent, **(Dest)** for destination, and **(Inst)** for instrument to dyadic relations, and mapping the type labels **Person**, **City**, and **Bus** to monadic relations:

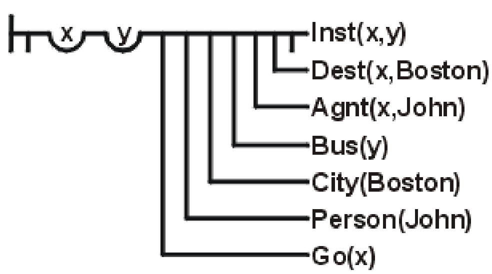
Peirce's algebraic notation (1885):

$$\Sigma_x \Sigma_y \, (\mathrm{Go}(x) \bullet \mathrm{Person(John)} \bullet \mathrm{City(Boston)} \bullet \mathrm{Bus}(y) \bullet$$
$$\mathrm{Agnt}(x,\mathrm{John}) \bullet \mathrm{Dest}(x,\mathrm{Boston}) \bullet \mathrm{Inst}(x,y))$$

With Peano's choice of symbols:

$$(\exists x)(\exists y)(\mathrm{Go}(x) \land \mathrm{Person(John)} \land \mathrm{City(Boston)} \land \mathrm{Bus}(y)$$
$$\land\ \mathrm{Agnt}(x,\mathrm{John}) \land \mathrm{Dest}(x,\mathrm{Boston}) \land \mathrm{Inst}(x,y))$$

Frege's Begriffsschrift for the Same Sentence



Translation to Peirce-Peano notation:

$$\sim(\forall x)(\forall y)(\mathrm{Go}(x) \supset (\mathrm{Person(John)} \supset (\mathrm{City(Boston)} \supset (\mathrm{Bus}(y) \supset$$
$$(\mathrm{Agnt}(x,\mathrm{John}) \supset (\mathrm{Dest}(x,\mathrm{Boston}) \supset \sim\mathrm{Inst}(x,y)))))))$$

Equivalent in English:

> *It is false that for every x and y, if x is an instance of going then if John is a person then if Boston is a city then if y is a bus then if the agent of x is John then if the destination of x is Boston then the instrument of x is not y.*

The Fate of Notations

Ernst Schröder wrote a negative review Frege's Begriffsschrift. Instead, he adopted Peirce's notation for his three volume *Vorlesungen über die Algebra der Logik*.

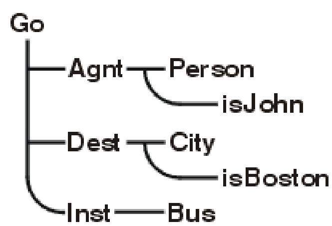
Hilbert, Zermelo, Löwenheim, and others adopted Peirce-Schröder notation for their early work on logic and set theory.

Peano adopted Peirce-Schröder notation, but changed the symbols.

Peano wrote a negative review of Frege's *Grundlagen der Arithmetik* to which Frege wrote a reply. In the correspondence, Peano insisted that Frege write all his examples in the algebraic notation.

Russell learned Peirce-Peano notation at a conference in 1900. By a judicious use of citations (or absence thereof), he managed to get the term *Peano-Russell notation* widely accepted.

Existential Graph for the Same Sentence



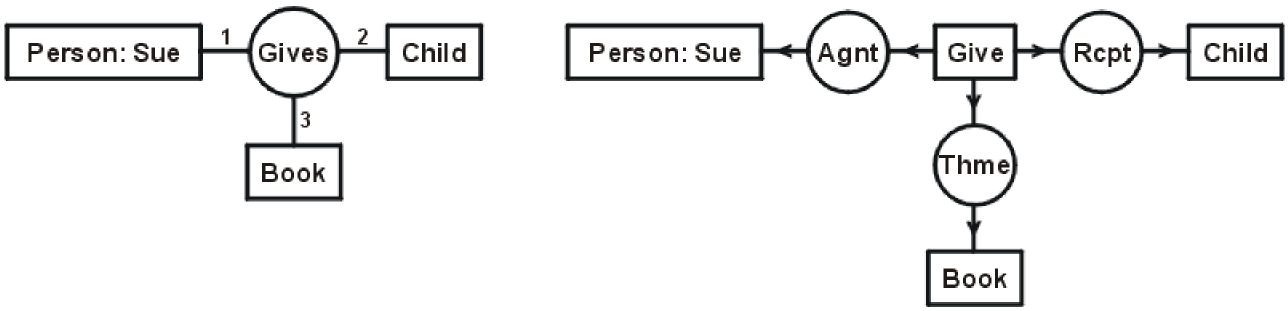
Equivalent in Peirce's algebraic notation:

$$\Sigma_x\Sigma_y\Sigma_z\Sigma_w\ (\mathrm{Go}(x) \bullet \mathrm{Person}(y) \bullet \mathrm{isJohn}(y) \bullet \mathrm{City}(z) \bullet \mathrm{isBoston}(z) \bullet \mathrm{Bus}(w) \bullet \mathrm{Agnt}(x,y) \bullet \mathrm{Dest}(x,z) \bullet \mathrm{Inst}(x,w))$$

Each bar or connected ligature of bars represents an existential quantifier. Each character string is the name of some relation. In existential graphs, Peirce's only primitives were relations, the existential quantifier, conjunction, and negation. Instead of names, he used monadic relations such as isBoston, which is true of only one individual.

Conceptual graphs are extensions of existential graphs in which the existential quantifier is represented by a concept node (drawn as a box), relations are represented by circles or ovals linked to the boxes, the type labels inside the boxes represent most monadic relations, and a name or other identifier can be written in the right side of any concept box. Other extensions are defined in terms of those primitives; a universal quantifier, for example, can also be written in the right side of a concept box (instead of a name or other identifier).

Irreducible Triad: *Sue gives a child a book.*



In CGs, a triadic relation such as **(Gives)** can be replaced by a concept **[Give]** with three attached arcs. The concept type **Give** is, in Peirce's terminology, a *hypostatic abstraction* of the relation type **Gives**. The idea of representing a verb by an entity that can be related by quantified variables is what Davidson called *event semantics*. Peirce's version of hypostatic abstraction is more general because it can also be used for relations derived from other parts of speech, such as adjectives and adverbs.

The equivalent operation can be performed in the algebraic notation, but its effect on the structure is harder to see and to express in a systematic generalization. Following is the predicate calculus representation for the CG on the left:

$$(\exists x)(\exists y)(\mathrm{Person}(\mathrm{Sue}) \wedge \mathrm{Child}(x) \wedge \mathrm{Book}(y) \wedge \mathrm{Gives}(\mathrm{Sue},x,y)).$$

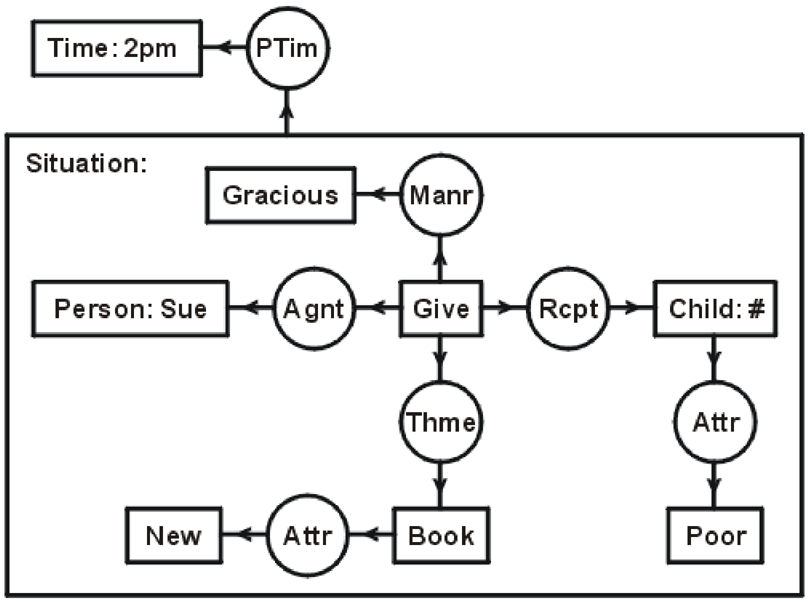
For the CG on the right, the triadic connection is represented by five occurrences of the variable z, three of which correspond to the three arcs attached to the concept **[Give].** The conceptual relations **(Rcpt)** for recipient and **(Thme)** for theme are translated to dyadic relations in predicate calculus:

$$(\exists x)(\exists y)(\exists z)(\mathrm{Person}(\mathrm{Sue}) \wedge \mathrm{Child}(x) \wedge \mathrm{Book}(y) \wedge \mathrm{Give}(z) \wedge \\ \mathrm{Agnt}(z,\mathrm{Sue}) \wedge \mathrm{Rcpt}(z,x) \wedge \mathrm{Thme}(z,y))$$

As this example illustrates, the graph notation directly shows the *topology* of the logic, which is determined by the connectivity of the nodes and the cycles in the graph. That same topology is present in the algebraic formulas, but it is obscured by the notation for variables and quantifiers. By showing the connections directly, the graph notation in either CG or EG form enables efficient graph operations that are difficult or impossible to apply to the formulas without first converting them to an equivalent graph. The importance of these operations is evident in the paper on analogical reasoning by Sowa and Majumdar (2003), and they are presented in more detail by Majumdar, Sowa, and Tarau (forthcoming).

Adding More Detail

Sentence: *At 2 pm, Sue graciously gave the poor child a new book.*



Following is the algebraic formula that corresponds to the conceptual graph:

$$(\exists s)(\text{Situation}(s) \land \text{Time}(2pm) \land \text{PTim}(s,2pm) \land \text{dscr}(s, (\exists y)(\exists z)(\exists u)(\exists v)(\exists w)(\text{Person}(Sue) \land \text{Child}(Bob) \land \text{Book}(y) \land \text{Give}(z) \land \text{Gracious}(u) \land \text{Poor}(v) \land \text{New}(w) \land \text{Manr}(z,u) \land \text{Attr}(Bob,v) \land \text{Attr}(y,w) \land \text{Agnt}(z,Sue) \land \text{Rcpt}(z,Bob) \land \text{Thme}(z,y)))).$$

The concept of type Situation with a nested CG represents a situation described by that CG. In the algebraic formula, the relation dscr(s,p) is used to state that a situation s is described by a proposition p. The relation **(Ptim)** shows the point in time of that situation. The relations **(Manr)** and **(Attr)** represent the manner and attribute relations that are linked to the hypostatic abstractions **[Gracious]** and **[Poor]**, which were derived from an adverb and an adjective in the original sentence. Those concepts represent instances of graciousness and poverty, and the graphs allow additional connections to those nodes to represent phrases such as *very graciously* or *poor as a church mouse*.

The symbol **#** in the concept **[Child: #]** represents the indexical effect of the phrase *the child*. Before the CG can be translated to other versions of logic, the indexical must be resolved to some individual in the context, either in the discourse or in the surrounding environment. In the algebraic formula, the symbol **#** is replaced by the name Bob. Hans Kamp developed discourse representation theory as a method of resolving such references. It turns out that the notation Kamp developed has context boxes that are isomorphic to the ovals of Peirce's existential graphs. By following Peirce's structures, the CG boxes turned out to be nested in the same ways as Kamp's.

Peirce’s Unified Framework

The previous examples illustrated several points:

1. Hypostatic abstraction is more general than Davidson’s event semantics.
2. Peirce’s ovals or CG boxes can demarcate situations, as in situation semantics.
3. Ovals or boxes correspond to the boxes in Kamp’s discourse representation theory.

There are many other features in Peirce’s work, which he represented in versions of existential graphs and which have been converted to conceptual graphs in forms that are consistent with the ongoing work in linguistics and logic. In the 20th century, many people reinvented aspects of Peirce’s vision:

1. Tarski: model theory and the importance of metalanguage.
2. Davidson: event semantics.
3. Austin: speech acts.
4. Grice: conversational implicatures.
5. Perry: the essential indexical.
6. Kamp: nested contexts for DRT.
7. Carnap, Kripke, Montague: possible worlds.

Many of these developments went into much more detail than Peirce had. But Peirce demonstrated that all these and many other aspects of language are part of a unified vision of language and logic.

Beyond Analytic Philosophy

Book by Hao Wang (1986).

Criticized the philosophy of his former thesis adviser:

> Quine merrily reduces mind to body, physical objects to (some of) the place-times, place-times to sets of sets of numbers, and numbers to sets. Hence, we arrive at a purified ontology which consists of sets only.... I believe I am not alone in feeling uncomfortable about these reductions. What common and garden consequences can we draw from such grand reductions? What hitherto concealed information do we get from them? Rather than being overwhelmed by the result, one is inclined to question the significance of the enterprise itself. (p. 146)

Quoted a personal letter from C. I. Lewis about philosophy in 1960:

> It is so easy... to get impressive ‘results’ by replacing the vaguer concepts which convey real meaning by virtue of common usage by pseudo precise concepts which are manipulable by ‘exact’ methods — the trouble being that nobody any longer knows whether anything actual or of practical import is being discussed. (p. 116)

Nothing Else vs. Something More

Wang, Lewis, Rescher, and many other philosophers observed a drastic narrowing of the scope of questions that the analytic philosophers considered legitimate.

Wang used the term "nothing more" to characterize their attitude of limiting the scope of topics to be explored and represented.

Frege (1879) set out "to break the domination of the word over the human spirit by laying bare the misconceptions that through the use of language often almost unavoidably arise concerning the relations between concepts."

If Carnap did not know how to formalize something, he dismissed it with the remark "That's poetry!"

Although Peirce and Whitehead were pioneers in logic, they were ignored by the analytic philosophers because they tried to address "something more" that was "not clear".

Whitehead's introduction of Russell for the William James lectures at Harvard:

> I am pleased to introduce my good friend Bertrand Russell. Bertie thinks that I am muddle-headed, but then, I think that he is simple-minded.

Poetry and Logic

The logician Alfred North Whitehead:

> Human knowledge is a process of approximation. In the focus of experience, there is comparative clarity. But the discrimination of this clarity leads into the penumbral background. There are always questions left over. The problem is to discriminate exactly what we know vaguely.

The poet Robert Frost:

> I've often said that every poem solves something for me in life. I go so far as to say that every poem is a momentary stay against the confusion of the world.... We rise out of disorder into order. And the poems I make are little bits of order.

The philosopher Ernst Cassirer:

> Like all the other symbolic forms, art is not the mere reproduction of a ready-made, given reality. It is one of the ways leading to an objective view of things and of human life. It is not an imitation but a discovery of reality.

Poetry and logic are complementary approaches to a common problem: developing patterns of symbols that capture important aspects of life in a memorable form.

Why is language ambiguous?

Three natural language design principles:

1. Learnable by infants without any formal training.

   ∴ Finite vocabulary

2. Able to express anything that anyone might ever want to say.

   ∴ Infinite extensibility

3. Accommodate human breathing and short-term memory.

   ∴ Short phrases

Result:

> The same stock of words is going to be used and reused.
>
> And lexical ambiguity is inevitable.

Why is language vague?

Lord Kelvin:

> Better a rough answer to the right question,
> than an exact answer to the wrong question.

Charles Sanders Peirce:

> It is easy to speak with precision upon a general theme. Only, one must commonly surrender all ambition to be certain. It is equally easy to be certain. One has only to be sufficiently vague. It is not so difficult to be pretty precise and fairly certain at once about a very narrow subject.

Ludwig Wittgenstein:

> Frege compares a concept to an area and says that an area with vague boundaries cannot be called an area at all. This presumably means that we cannot do anything with it. — But is it senseless to say: "Stand roughly there"?

Limits of Definability

- Immanuel Kant:

  "Since the synthesis of empirical concepts is not arbitrary but based on experience, and as such can never be complete (for in experience ever new characteristics of the concept can be discovered), empirical concepts cannot be defined.

  "Thus only arbitrarily made concepts can be defined synthetically. Such definitions... could also be called *declarations*, since in them one declares one's thoughts or renders account of what one understands by a word. This is the case with *mathematicians*."

- Wittgenstein's *family resemblance*:

  Empirical concepts cannot be defined by a fixed set of necessary and sufficient conditions. Instead, they can only be taught by giving a series of examples and saying "These things and everything that resembles them are instances of the concept."

- Waismann's *open texture*:

  For any proposed definition of empirical concepts, new instances will arise that "obviously" belong to the category but are excluded by the definition.

Limits of Logic

Alfred North Whitehead, *Modes of Thought*:

- "Both in science and in logic, you have only to develop your argument sufficiently, and sooner or later you are bound to arrive at a contradiction, either internally within the argument, or externally in its reference to fact."
- "The topic of every science is an abstraction from the full concrete happenings of nature. But every abstraction neglects the influx of the factors omitted into the factors retained."
- "The premises are conceived in the simplicity of their individual isolation. But there can be no logical test for the possibility that deductive procedure, leading to the elaboration of compositions, may introduce into relevance considerations from which the primitive notions of the topic have been abstracted."

Summary: "We must be systematic, but we should keep our systems open."

Microsenses

A term coined by Alan Cruse for the infinite variability of word senses.

The word "car" —

- 1900: A horeseless carriage, complete with hooks to attach a real horse when it breaks down.
- Today: A metal cabin with more computing power than the supercomputers of the 1980s.
- Tomorrow: Acronym for Computerized Autonomous Rover.

The word "file" —

- IBM mainframe: A bit string separated into records by the operating system.
- Unix: A character string separated by new-line characters.
- Macintosh: A character string separated by carriage-return characters.
- Windows: A character string separated by new-line plus carriage-return.

Formal definitions change with every release and patch to the software.

Challenge to Artificial Intelligence

Position paper by Alan Bundy, pioneer in automated problem solving:

- A few minutes studying any particular representation rapidly reveals deficiencies in expressivity or efficiency or both.
- The world is infinitely complex, so there is no end to the qualifications, ramifications and richness of detail that one could incorporate, and that you might need to incorporate for a particular application.
- For a narrow application, it is often sufficient to hand-craft a representation that hits the desired sweet spot.
- In general, the representation itself needs to be manipulated automatically.
- Such manipulation must be able to change the underlying syntax and semantics of the ontology.
- We believe that automatic representation development, evolution and repair must be a major goal of artificial intelligence research over the next 50 years.

These points have been extracted from the following paper:

Bundy, Alan, & Fiona McNeill (2006) “Representation as a Fluent: An AI Challenge for the Next Half Century,” AAAI Fellows Symposium, Boston, MA.

Peirce's Conclusion

At the end of the 19th century, Peirce was employed as an associate editor of the *Century Dictionary*.

He wrote, revised, or edited over 16,000 definitions — more than any other editor of that dictionary.

In a letter to the general editor, B. E. Smith, he expressed his observations about logic and language:

> The task of classifying all the words of language, or what's the same thing, all the ideas that seek expression, is the most stupendous of logical tasks. Anybody but the most accomplished logician must break down in it utterly; and even for the strongest man, it is the severest possible tax on the logical equipment and faculty.

Since they were creating an unabridged dictionary, Peirce and his colleagues were forced to define every word that occurred in their corpus of citations.

Unlike Frege and Carnap, Peirce had to deal with language as it is, not as a logician might wish it to be.

Suggested Readings

The paper with the same title that appeared in the ICCS'06 proceedings:

http://www.jfsowa.com/pubs/csp21st.pdf

A talk, "Concept Mapping," which discusses issues of ambiguity and vagueness from a related point of view:

http://www.jfsowa.com/talks/cmapping.pdf

Two papers that show the importance of algorithms that take advantage of graph structure:

Sowa, John F., & Arun K. Majumdar (2003) "Analogical reasoning," in A. de Moor, W. Lex, & B. Ganter, eds., *Conceptual Structures for Knowledge Creation and Communication*, LNAI 2746, Springer-Verlag, Berlin, pp. 16-36. http://www.jfsowa.com/pubs/analog.htm

Sowa, John F. Sowa (2008) "Conceptual Graphs," in F. van Harmelen, V. Lifschitz, and B. Porter, eds., *Handbook of Knowledge Representation*, Elsevier, 2008, pp. 213-237.

Two papers that adapt some of Peirce's ideas to the semantics of modal logic and the interpretation of possible worlds in terms of Michael Dunn's semantics of laws and facts:

Sowa, John F. (2003) "Laws, facts, and contexts: Foundations for multimodal reasoning," in *Knowledge Contributors*, edited by V. F. Hendricks, K. F. Jørgensen, and S. A. Pedersen, Kluwer Academic Publishers, pp. 145-184. http://www.jfsowa.com/pubs/laws.htm

Sowa, John F. (2006) "Worlds, Models, and Descriptions," *Studia Logica*, Special Issue, *Ways of Worlds II*, to appear in November, 2006.