

Logic, Ontology, and Analogy

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The Problem of Knowledge Representation

As stated by the logician Alfred North Whitehead:

Human knowledge is a process of approximation. In the focus of experience, there is comparative clarity. But the discrimination of this clarity leads into the penumbral background. There are always questions left over. The problem is to discriminate exactly what we know vaguely.

And by the poet Robert Frost:

I've often said that every poem solves something for me in life. I go so far as to say that every poem is a momentary stay against the confusion of the world.... We rise out of disorder into order. And the poems I make are little bits of order.

Poetry and logic are complementary approaches to a common problem:

Create patterns of symbols that capture important aspects of life in a memorable form.

But those patterns of symbols, in logic or in poetry, are approximations.

Origin of Formal Logic and Ontology

The ancient Greeks had as many lawsuits as modern Americans.

But the plaintiffs and defendants had to plead their own cases in court.

The Sophists earned a good living by teaching people how to present their cases in the best possible light.

Plato criticized the Sophists for ignoring truth and trying to make the weaker case seem to be the stronger.

Aristotle analyzed and classified the arguments:

- * Deductive methods preserve truth.
- * Unsound methods, called fallacies, are plausible, but misleading.
- * Syllogisms are deductive methods for reasoning about the categories of an ontology.
- * Original meaning of κατηγορία: an accusation in a court of law.

Prospects for a Universal Ontology

- * 4th century BC: Aristotle's categories and syllogisms.
- * 17th century: Universal language schemes by Descartes, Mersenne, Pascal, Leibniz, Newton, Wilkins, and others.
- * 18th century: More schemes, the Grand Academy of Lagado, Kant's categories.
- * 19th century: Roget's Thesaurus, Oxford English Dictionary.
- * Early 20th century: Many terminologies in many different fields.
- * 1960s: Computerized versions of the terminologies.
- * 1970s: ANSI/SPARC Conceptual Schema.
- * 1980s: Cyc, WordNet, Japanese Electronic Dictionary project.
- * 1990s: SRKB, ISO Conceptual Schema, Semantic Web, many workshops.
- * 2000s: Many proposals, no consensus.

Informal terminologies and dictionaries have been extremely successful.

Formal systems are still research projects.

Notations for Representing Knowledge

Writing a precise statement of knowledge in any language, even one's own native language, is not easy.

As Whitehead said, “the problem is to discriminate precisely what we know vaguely.”

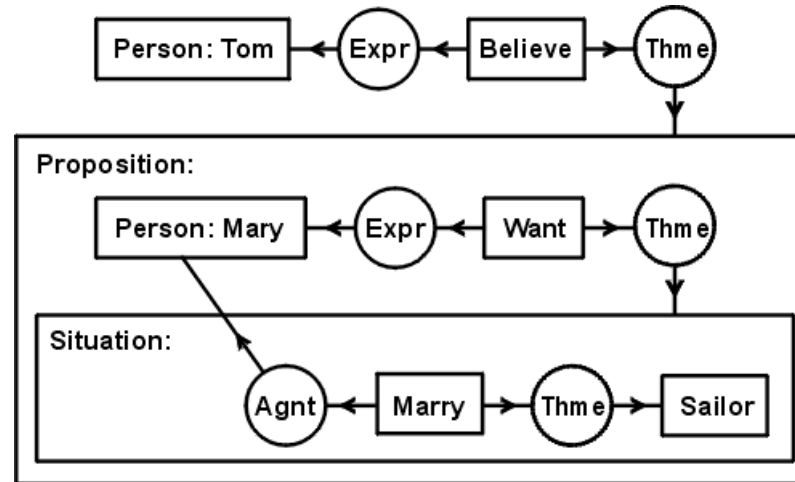
Informal notations are useful, but many steps are needed to convert them to a formal specification.

Controlled English, as in CLCE, is easy for humans to read, but humans require considerable training before they can write it.

Conceptual graphs are an intermediate notation that can be used in informal methods that can be made precise by systematic steps.

Example: *Tom believes Mary wants to marry a sailor.*

Conceptual graph:



Conceptual Graph Interchange Format (CGIF):

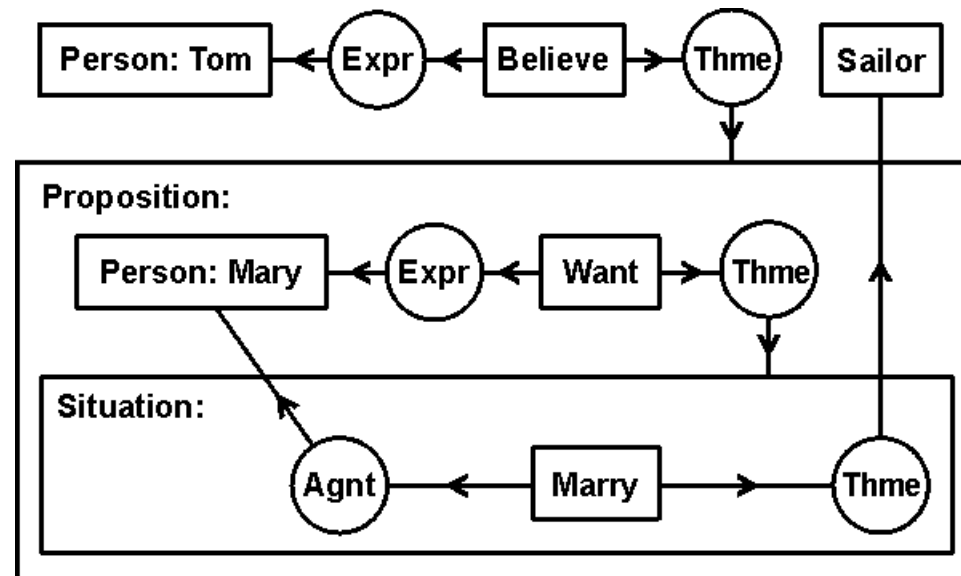
```
[Person *x1 Tom] [Believe *x2] (Expr ?x2 ?x1)
  (Thme ?x2 [Proposition
    [Person *x3 Mary] [Want *x4] (Expr ?x4 ?x3)
    (Thme ?x4 [Situation
      [Marry *x5] (Agnt ?x5 ?x3) (Thme ?x5 [Sailor]) ] ] ])
```

Common Logic Interchange Format (CLIF):

```
(exists ((x1 Person) (x Believe)) (and (= x1 Tom) (Expr x2 x1)
  (thme x2 (that (exists ((x3 Person) (x4 Want) (x8 Situation))
    (and (= x3 Mary) (Expr x4 x3) (Thme x4 x8)
      (Dscr x8 (that (exists ((x5 Marry) (x6 Sailor))
        (and (Agnt x5 x3) (Thme x5 x6))))))))))
```

There is a sailor that Tom believes Mary wants to marry.

Conceptual graph:



CGIF:

```
[Person *x1 Tom] [Sailor *x6] [Believe *x2] (Expr ?x2 ?x1)
  (Thme ?x2 [Proposition:
    [Person *x3 Mary] [Want *x4] (Expr ?x4 ?x3)
    (Thme ?x4 [Situation
      [Marry *x5] (Agnt ?x5 ?x3) (Thme ?x5 ?x6) ] ] ])
```

CLIF:

```
(exists ((x1 Person) (x6 Sailor) (x2 Believe))
  (and (= x1 Tom) (Expr x2 x1)
    (Thme x2 (that (exists ((x3 Person) (x4 Want) (x8 Situation))
      (and (= x3 Mary) (Expr x4 x3) (Thme x4 x8)
        (Dscr x8 (that (exists ((x5 Marry)
          (and (Agnt x5 x3) (Thme x5 x6))))))))))))))
```

Methods of Reasoning

Three methods of formal logic:

1. Deduction. Apply a general principle to infer some fact.

Given: Every bird flies. Tweety is a bird.
Infer: Tweety flies.

2. Induction. Assume a general principle that subsumes many facts.

Given: Tweety, Polly, and Hooty are birds.
Tweety, Polly, and Hooty fly.
Assume: Every bird flies.

3. Abduction. Guess a new hypothesis that explains some fact.

Given: Every bird flies. Tweety flies.
Guess: Tweety is a bird.

According to Peirce (1902), *“Besides these three types of reasoning there is a fourth, analogy, which combines the characters of the three, yet cannot be adequately represented as composite.”*

Four Views of Analogical Reasoning

1. By logicians:

Deduction is reasoning from “first principles.”

Analogy is an unsound, but interesting heuristic.

2. By psychologists:

Analogy is a fundamental cognitive mechanism.

Language and reasoning depend heavily on analogy.

3. Theoretical:

Analogy is a very general pattern-matching process.

Deduction, induction, and abduction depend on disciplined uses of analogy.

4. Computational:

A powerful and flexible technique for reasoning, learning, and language processing.

But practicality depends on finding analogies efficiently.

Example of Analogy: *How is a cat like a car?*

Analogy of Cat to Car	
Cat	Car
head	hood
eye	headlight
cornea	glass plate
mouth	fuel cap
stomach	fuel tank
bowel	combustion chamber
anus	exhaust pipe
skeleton	chassis
heart	engine
paw	wheel
fur	paint

Data from WordNet and other sources are translated to conceptual graphs.

The VivoMind Analogy Engine (VAE) starts at the nodes for Cat and Car and tries to find the longest matching paths through the graphs.

All analogies found are ranked by a semantic-distance measure. The above analogy received the highest score for that pair of words

Operations Performed During Pattern Matching

Following paths from each starting node:

[Cat]→(HasPart)→[Head]→(HasPart)→[Eye]→(HasPart)→[Cornea]

[Car]→(HasPart)→[Hood]→(HasPart)→[Headlight]→(HasPart)→[GlassPlate]

Matching concept nodes with similar types, properties, and relations:

- Head and hood are in the front.
- Eyes and headlights are related to light.
- Cornea and glass plate are transparent.
- Paws and wheels support the body, and there are four of each.

Approximate matching (ignoring the nodes in red):

[Cat]→(HasPart)→[Mouth]→(Flow)→[Esophagus]→(Flow)→[Stomach]→
→(Flow)→[Bowel]→(Flow)→[Anus]

[Car]→(HasPart)→[FuelCap]→(Flow)→[FuelTank]→
→(Flow)→[CombustionChamber]→(Flow)→[Muffler]→(Flow)→[ExhaustPipe]

Note: The source data did not contain information about the pipe from the fuel cap to the fuel tank. That missing part would have matched the esophagus of the cat.

Three Methods Used by VAE

- 1. Matching concept types, in order of increasing semantic distance:**
 - Identical types.
 - Subtype - supertype.
 - Siblings of same supertype.
 - More distant cousins.
- 2. Matching subgraphs:**
 - Match isomorphic subgraphs independent of type labels.
 - Merge adjacent nodes to make them isomorphic.
- 3. Finding metalevel mappings that can relate subgraphs,
— even though they are not isomorphic.**

Methods #1 and #2 are used for the Cat-Car example.

Method #3 (combined with #1 and #2) is used for aligning ontologies.

Criticisms of Logical Deduction

Deduction in mathematics can be precise, but deduction about any empirical subject must depend on prior induction, which is almost always incomplete.

Criticism by the physician, Sextus Empiricus (2nd century AD):

Every human is an animal.

Socrates is human.

Therefore, Socrates is an animal.

If the major premise was derived by checking every human, then Socrates was considered, and the argument is circular.

Otherwise, the induction was incomplete, and the conclusion is uncertain.

Criticism by the legal scholar, Ibn Taymiyya (14th century AD):

Every empirical theory is derived by induction from cases.

Any deduction from such a theory can be obtained by applying analogical reasoning to exactly the same cases.

Example of Case-Based Reasoning

Reasoning by cases is essential in business, medicine, and law.

Example by Ibn Taymiyya:

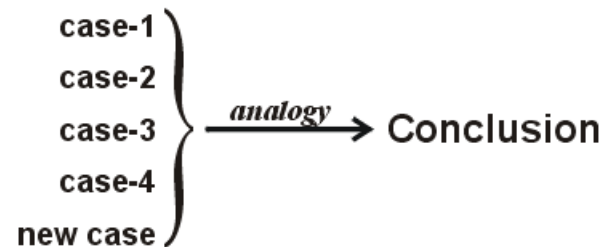
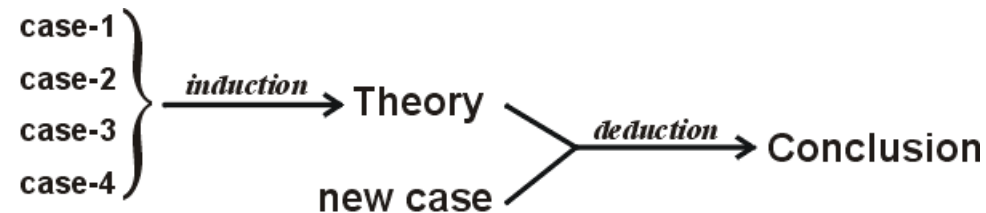
1. **Given case:** Grape wine is prohibited.
2. **Question:** Is date wine prohibited?
3. **Cause:** Grape wine is prohibited because it is intoxicating.
Date wine is also intoxicating.
4. **Judgment:** Date wine is also prohibited.

More complex cases have multiple aspects and mitigating circumstances.

But any distinctions that can be formalized in a theory can also be discovered and applied by means of analogy.

Reasoning by Cases and by Theory

A diagram of Ibn Taymiyya's argument:



If the same cases are used, analogy can give the same answer in one step that requires two steps by induction followed by deduction.

What Ibn Taymiyya did not recognize: If the same theory can be reused for many different applications, deduction is more efficient.

Logic as a Disciplined Use of Analogies

The structure-mapping operations of analogy are used in every kind of logical reasoning:

- * Deduction. Every step requires a unification, which is special case of the structure mappings used in analogies.**
- * Induction. Analogies are used to find common generalizations of multiple instances.**
- * Abduction. The operation of guessing or forming an initial hypothesis, called abduction, requires analogies to find likely causes or explanations.**

In both human reasoning and computer implementations, the same underlying operations can support both logical and analogical reasoning.

Application to Educational Software

A textbook publisher needs to evaluate student answers to math questions.

- * Free-form answers in English sentences.
- * Much harder to evaluate than multiple choice.

Typical question:

The following numbers are 1 more than a square: 10, 37, 65, 82.

*If you are given an integer N that is less than 200,
how would you determine whether N is 1 more than a square?*

Explain your method in three or four sentences.

How could a computer system evaluate student answers?

Determine whether they are correct, incorrect, or partially correct?

And make helpful suggestions about the incorrect answers?

Publisher's Current Procedure

To evaluate new exam questions, the publisher normally gives the exam to a large number of students.

For each question, they get about 50 different answers:

- * Some are completely correct
— but stated in different ways.**
- * Some are partially correct
— and the teacher says what is missing.**
- * Others are wrong
— in many different ways.**

Result: 50 pairs of student answer and teacher response.

Each pair of (answer,response) is a case for case-based reasoning.

Three Teams Addressed this Problem

Team #1 used a large deductive knowledge base.

- * Ontology for mathematical word problems.**
- * English parser to analyze student answers.**
- * Theorem prover to determine if the answers are correct.**

Team #2 used Latent Semantic Analysis (LSA).

- * Measure the similarity of student answers to correct answers.**

VivoMind team used case-based reasoning.

- * Intellitex to translate student answers to conceptual graphs.**
- * VAE to compare student CGs to correct and incorrect CGs.**

Results for Team #1

Extending the ontology for each problem type was too difficult for most high-school teachers.

Student answers had too many fragmentary and ungrammatical sentences to be parsed correctly.

The theorem prover could not reach a conclusion for most of the student answers.

The method was considered impractical.

Results for Team #2

LSA methods usually give good results for measuring the similarity of texts that are longer than a single paragraph.

They are less reliable on texts of just a few sentences.

They cannot discriminate between answers that interchange words or insert an extra word, such as 'not'.

They were unreliable for distinguishing correct answers from incorrect answers.

Intellitex Parser

VivoMind has developed a parser named Intellitex:

- * Uses a rather simple grammar.**
- * Depends on analogies for interpreting sentences.**
- * Generates conceptual graphs as output.**
- * Robust: always generates some CG as its best guess.**

These properties are important for handling typical student answers, which frequently have poor grammar and incomplete sentences.

Minor errors are not necessarily bad — provided that Intellitex makes the same errors consistently in all cases.

Using VAE to Evaluate Student Answers

Use VAE to compare each new answer to the 50 cases:

1. For all 50 cases, translate student answer to conceptual graphs.
2. Translate each new answer to a new CG.
3. Compare the new CG to the 50 CGs for previous answers.
4. Use the measure of semantic distance to determine the best match.
5. If there is a good match, print out the corresponding response.
6. Otherwise, send the new answer to a teacher to evaluate.

Result:

VAE found a good match for nearly all of the student answers.

For each good match, the previous teacher's response was appropriate.

When VAE failed to find a good match, the new case could be added to the list of cases in order to improve its coverage.

There is no need for the teachers to write rules, programs, or ontologies.

Conclusions

No evidence of formal logic as a prerequisite for learning, understanding, or speaking a natural language.

Common logical operators — *and, or, not, if-then, some, every* — are present in every NL. But they are used in many different senses, which include classical first-order logic as an important special case.

Reasoning by analogy is fundamental. Induction, deduction, and abduction are important, highly disciplined special cases.

But analogy is a more general reasoning method, which can be used even with images, prior to any version of language.

No evidence of a highly axiomatized ontology for any natural language.

But many important commonalities result from common human nature, experience, and activities.

Formal, logic-based systems with deeply axiomatized ontologies have been fragile and limited in their coverage of natural language texts.

Analogy-based systems with loosely defined terminologies can be far more robust and efficient for many applications.